Quotient Rule

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

The product rule is used to differentiate an expression where one term is divided by another. This formula means that if we differentiate the value $\frac{u}{v}$ then that will be equal to v ×(derivative of u) - u ×(derivative of v)[1] all divided by v^2.

For example, if we have the expression $y = \frac{x-6}{3x-1}$ and we want to differentiate this.

We can say that $u = (x-6)$ and $v = (3x-1)$.
So

$$\frac{d}{dx}\frac{x-6}{3x-1} = \frac{(3x-1)\frac{d}{dx}(x-6) - (x-6)\frac{d}{dx}(3x-1)}{(3x-1)^2}$$
$$\frac{d}{dx}\frac{x-6}{3x-1} = \frac{(3x-1)\times 1 - (x-6)\times 3}{(3x-1)^2}$$
$$\frac{d}{dx}\frac{x-6}{3x-1} = \frac{3x-1-3x+18}{(3x-1)^2}$$
$$\frac{d}{dx}\frac{x-6}{3x-1} = \frac{17}{(3x-1)^2}$$

Proof

Let $y = \frac{u}{v}$ where u and v are functions of x. If we increase x by δx (δx is a very small amount), then u and v increase by δu and δv and so y increases by δy
This means that

$$y + \delta y = \frac{u + \delta u}{v + \delta v}$$

But we know $y = \frac{u}{v}$, so subtracting that

$$(y + \delta y) - y = \left(\frac{u + \delta u}{v + \delta v}\right) - \frac{u}{v}$$

This can be expressed in the form

$$\delta y = \frac{v(u + \delta u)}{v(v + \delta v)} - \frac{u(v + \delta v)}{v(v + \delta v)}$$

So

$$\delta y = \frac{uv + v\delta u - uv - u\delta v}{v^2 + v\delta v}$$

Which can be simplified to

$$\delta y = \frac{v\delta u - u\delta v}{v^2 + v\delta v}$$

'Dividing' by δx

[1] The derivative is the expression we get as the result of differentiation

$$\frac{\delta y}{\delta x} = \frac{v\frac{\delta u}{\delta x} - u\frac{\delta v}{\delta x}}{v^2 + v\delta v}$$

As $\delta x \to 0$ and $\delta v \to 0$
and

$$\frac{\delta y}{\delta x}, \frac{\delta v}{\delta x}, \frac{\delta u}{\delta x} \to \frac{dy}{dx}, \frac{dv}{dx}, \frac{du}{dx}$$

Then

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

<u>See also</u>
- Product Rule

<u>References</u>
Turner, L. K. (1976). *Advanced Mathematics – Book One*. London: Longman. pp.113-115.